UNDERSTANDING MODAL PARAMETER TECHNOLOGY AND MODE SHAPE SCALING

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ABSTRACT

Modal analysis has become a frequently used method for studying the dynamic behavior of mechanical structures. This technique is usually presented using parameters which are defined by complicated mathematical derivations. Unfortunately, it is difficult for the experimentalist to apply these mathematical concepts to the solution of his vibration problem. Consequently, modal analysis has become a "black box". technique with frequency response measurements as input and mode shapes as the output. To examine this "Black box", and in particular mode shape scaling, we present an illustrative example that relates both the terminology and mathematical concepts to practical applications.

INTRODUCTION

For our example, we will determine the modal parameters of a set of measurements which were acquired from the structure shown in Figure 1. To better exemplify an actual modal test we will assume that the physical properties of the test structure (eg. mass, stiffness) are unknown. Because mode shape scaling is critical to further data analysis (eg. structural modification), it will be explained in detail.

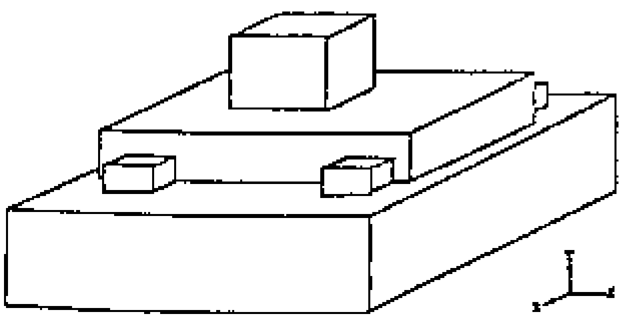

FIGURE 1

Although the structure is a continuous system, we will model it as a system of three discrete elements. For simplicity, we will assume the vibratory motion is in the ydirection only. This simplifies the model to the three degreeoffreedom system shown in Figure 2.

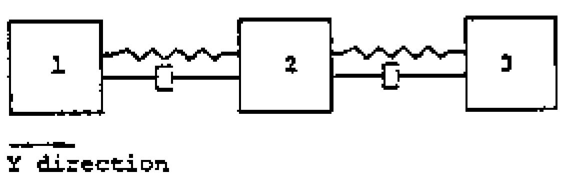

FIGURE 2

FREQUENCY RESPONSE MEASUREMENTS

Because we are modeling the test structure as a threedegreeoffreedom system, nine frequency response measurements completely define its dynamic characteristics. For our example, we will assume that the frequency response data has already been gathered and that the response units are inches and the force units are pounds. Figure 3 is an illustration of the force and response transducer positions and Figure 4 is a matrix of the measurements acquired from those configurations.

FREQUENCY AND DAMPING

We can now extract the modal parameters from the frequency response measurements. The frequency and damping must be determined first. We will use a measurement in which the peaks are well defined, such as measurement 1y/1y shown in Figure 5.

The measurement in Figure 5 contains three peaks. These peaks correspond to the three modes of the structure. The frequency and damping of each of these modes was found using standard curve fitting techniques and are listed in the Table 1.

Frequency and damping are global properties. This means that they do not vary across the structure, and can be estimated from any frequency response measurement taken from the structure except those measured at any point where the mode shape has zero amplitude.

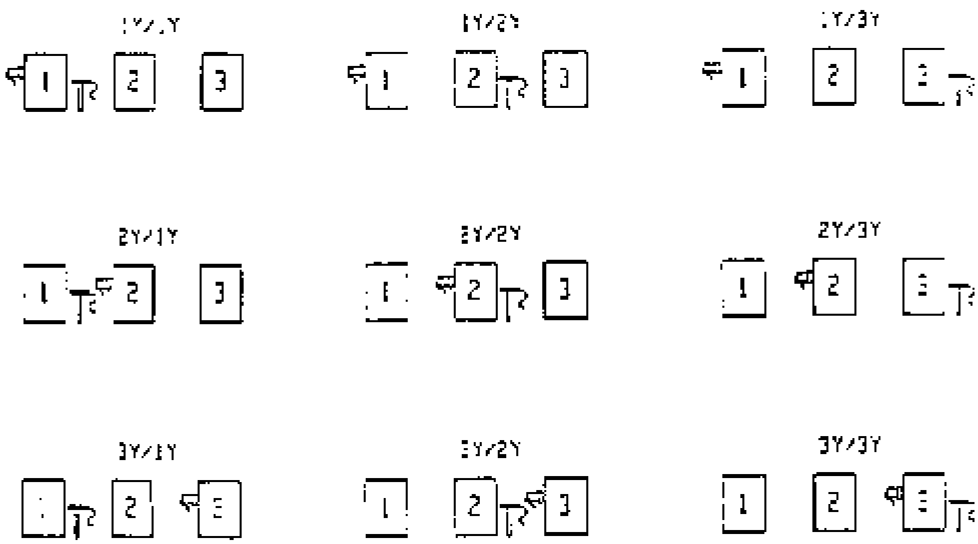

FIGURE 3

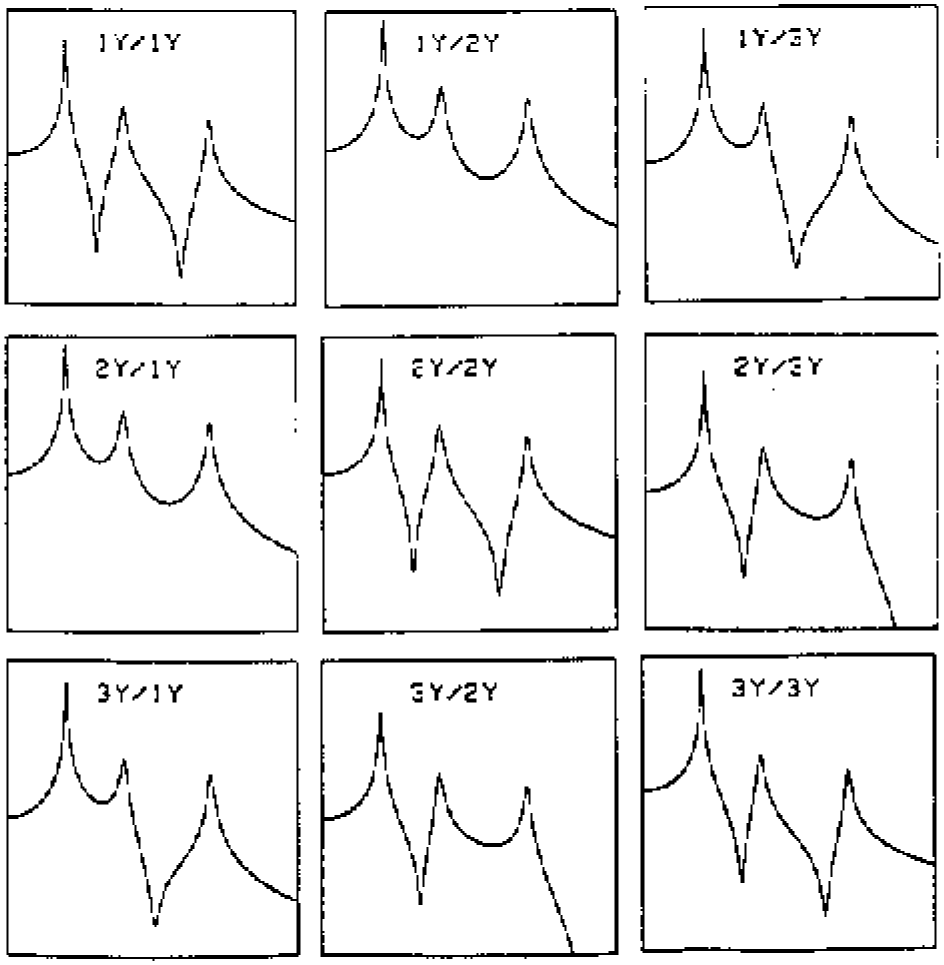

FIGURE 4

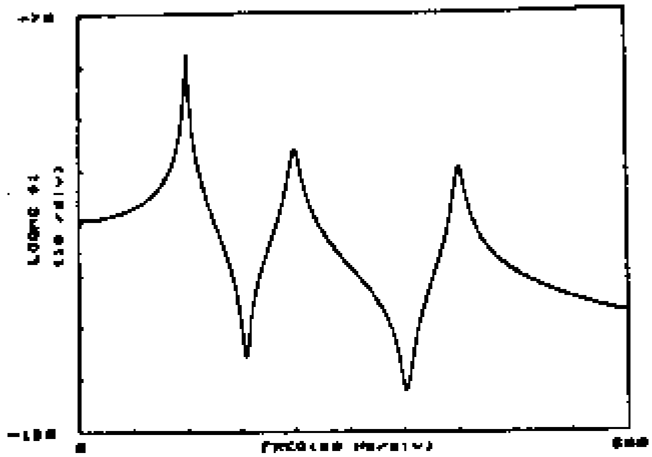

FIGURE 5

MODE NO.	FREQUENCY (Hz)	DAMPING (%)
1	100.00	1.00
2	200.00	2.00
3	350.00	1.00

TABLE 1

MODE SHAPES

Now, we will use the amplitude of the peaks to determine the mode shapes. The amplitude of the first peak of each measurement in Figure 4 is listed in Figure 6.

AMPLITUDES OF THE
FREQUENCY RESPONSE MEASUREMENTS
FOR MODE 1

1y/1y 1.267E-04	1y/2y 2.533E-04	1y/3y 5.065E-04
2y/1y 2.533E-04	2y/2y 5.067E-04	2y/3y 10.13E-04
3y/1y 5.065E-04	3y/2y 10.13E-04	3y/3y 20.26E-04

in/lb_f

FIGURE 6

Scaling the three columns of the above matrix such that the smallest element of each column is equal to one (ie. dividing each element of a column by the smallest element of that column), yields the modal vectors shown in Figure 7.

SCALED MODE SHAPES FOR MODE 1

$$\begin{Bmatrix} 1.0 \\ 2.0 \\ 4.0 \end{Bmatrix} \qquad \begin{Bmatrix} 1.0 \\ 2.0 \\ 4.0 \end{Bmatrix} \qquad \begin{Bmatrix} 1.0 \\ 2.0 \\ 4.0 \end{Bmatrix}$$

FIGURE 7

We can also scale the rows in a similar manner and obtain a matrix with the mode shape (1, 2, 4) in each row. We can see from this scaling exercise that the complete mode shape is defined in every column and row. Given this and recalling that the frequency and damping can be determined from any measurement, we can conclude that we need only measure one row or one column of the frequency response matrix to completely define all the modal parameters.

Now referring to Figure 3, we can describe this property as it relates to a modal test. A column of the frequency response matrix contains measurements acquired by fixing the location of the input force and moving the response to each test point. Similarly, a row contains measurements acquired by fixing the response location and moving the input to each test point.

The ramifications of this property of the frequency response matrix are that we need not measure all nine frequency response measurements to obtain the modal parameters. By selecting one exciter location and measuring the response at all three points or by fixing the response location and exciting the structure at all three points, we can obtain all of the structure's modal parameters.

From this point on we will assume that we only have data for the first column of the frequency response matrix. This means that we have taken data with the exciter located at mass #1 and the response measured at all three masses. Now let's take a closer look at the scaling of the mode shapes.

The mode shape in Figure 8 is scaled but uncalibrated. This means that it is scaled to a convenient format (in this case, the convenience is for ease of reading) but not calibrated to the frequency response measurements. This mode shape can be used for plotting or animated display.

The amplitudes in Figure 9 are an unscaled but calibrated mode shape. This means that the mode shape values are directly related to the frequency response measurements but are not scaled to a convenient format.

SCALED MODE SHAPE FOR MODE 1

$$\begin{Bmatrix} 1.0 \\ 2.0 \\ 4.0 \end{Bmatrix}$$

FIGURE 8

CALIBRATED MODE SHAPE FOR MODE 1

$$\begin{Bmatrix} 1.267E-04 \\ 2.533E-04 \\ 5.065E-04 \end{Bmatrix} \; in/lb_f$$

FIGURE 9

For users that require only frequency, damping, and a display of the mode shape, an uncalibrated mode shape (ie. Figure 8) is sufficient, regardless of how it is scaled. However, for advanced analysis techniques such as structural dynamics modification, the mode shape must be calibrated and scaled so that the correct inertial and elastic properties of the structure are preserved. In our example, the mode shapes will be calibrated and scaled using two techniques.

MODE SHAPE SCALING

The first scaling technique is based on an expression which relates the peak values of the frequency response measurements to a scaled calibrated mode shape. This expression, known as a residue, is related to the frequency response measurements by a term proportional to the damping at the natural frequency. The residues were computed from our example data using a standard curve fitting method, and those for mode #1 are shown in Figure 10.

RESIDUES FOR MODE 1

$$\begin{Bmatrix} 1y/1y \\ 1.592E-03 \\ 2y/1y \\ 3.183E-03 \\ 3y/1y \\ 6.367E-03 \end{Bmatrix} \; in/lb_f\text{-sec}$$

FIGURE 10

SCALED MODE SHAPE FOR MODE 1

$$\begin{Bmatrix} 1 \\ 2 \\ 4 \end{Bmatrix}$$

FIGURE 11

Scaling the residues such that the smallest element is equal to one (ie. dividing each element by the smallest element) yields the same mode shape as that obtained previously (See Figure 11). This shows that the unscaled calibrated mode shape can be either the amplitudes of the frequency response measurements or the residues.

The advantage of using the residues is that they provide a convenient way to scale the mode shapes while retaining the calibration. If we look closely at the residues we see that each one is actually comprised of three values; a calibration constant, and two coefficients which provide the scaling. The composition of the residues, for mode 1 of our example, is shown in Figure 12.

$$\begin{Bmatrix} R_{1y/1y} \\ R_{2y/1y} \\ R_{3y/1y} \end{Bmatrix} = \frac{1}{M^{*}w_d} \begin{Bmatrix} U_{1y}{}^{*}U_{1y} \\ U_{2y}{}^{*}U_{1y} \\ U_{3y}{}^{*}U_{1y} \end{Bmatrix} \qquad (1)$$

where,

R = Residues computed from column one of the frequency reponse measurements.
w_d = Natural frequency for mode 1.
M = Scaling constant for the mode shape.
U_{1y} = Coefficient related to location 1y.
U_{2y} = Coefficient related to location 2y.
U_{3y} = Coefficient related to location 3y.

FIGURE 12

UNITY MODAL MASS SCALING

In general, each equation in Figure 12 contains three unknowns. In the third equation, for example, M, U_{1y} and U_{3y} are unknown while the residue $R_{1y}/_{1y}$ and the natural frequency wd are known. However, if a measurement is taken at a "driving point". (ie. the force and response measured at the same location), the coefficients are repeated, thus reducing the number of unknowns to two for that particular equation. The driving point which we will use in our example is measurement $1_y/1_y$. Figure 13 shows that the first equation has only two unknowns, M and U_{1y}.

$$R_{1y/1y} = \frac{1}{M^{*}w_d} U_{1y}{}^{*}U_{1y} \qquad (2)$$

If we solve for U1y we have the solution in terms of the residue $R_{1y}/_{1y}$ the natural frequency wd, and a scaling constant M. Now, if we arbitrarily choose the scaling constant M to be unity, then we can solve for U_{1y}. Once U_{1y} is known, the other mode shape coefficients can be determined. These coefficients comprise the scaled calibrated mode shape.

In Figure 13, the scaling constant M is known as the "Modal mass". For the single degreeof-freedom case, the mode shape coefficient U_{1y} is equal to one (1) and the expression for modal mass is the actual mass of the structure. Of course, this is a theoretical case and in the actual case the value of M is related to how the mode shape is scaled. In our case we arbitrarily set the value of M to unity and scale the mode shape accordingly. This technique is known as unity modal mass scaling.

SCALING THE MODE SHAPES USING EFFECTIVE MASS/STIFFNESS

In the second technique, the mode shape scaling is completely arbitrary. This means that the mode shape itself does not contain any calibration information. Rather, a calibration constant is computed for a particular measurement and mode. To demonstrate this, we will use the mode shape shown in Figure 8. Recall that this mode shape is scaled but not calibrated. To calibrate this mode shape we will compute the residue for a particular point and tag it to the corresponding degree of freedom.

The residue for mode #1 of measurement $1_y/1_y$, computed using a standard curve fitting method, is 1.592E03 in/lbf. Since we know the relative amplitudes for the rest of the mode shape, this residue provides the calibration for the entire mode shape. Therefore, by calculating the residue for one test point and tying it to one value of the mode shape, we retain the calibration for the entire mode shape. Recall that for the single decree of freedom case, the mode shape coefficients in equation 2 are equal to one. Rearranging this reduced equation gives the modal mass in terms of the residue and natural frequency only (equation 3).

$$M_{1y/1y} = \frac{1}{w_d * R_{1y/1y}} \quad (3)$$

$$K_{1y/1y} = \frac{w_d}{R_{1y/1y}} \quad (4)$$

Using the relationship, w^2 = K/m, together with equation 3 allows us to also represent the modal stiffness in terms of the natural frequency and residue (equation 4). Like modal mass, the term modal stiffness originated from the single degreeoffreedom model. For the multiple degreeoffreedom case these values are the calibration for the mode shape. Some users have been able to use this value when approximating a single degreeoffreedom case, hence they are also called effective mass and the effective stiffness. This approximation is highly dependent on the type of structure being tested and in not normally made.

SUMMARY

This parameter estimation example was presented to explain the modal analysis process following the acquisition of the frequency response measurements. By closely examining the frequency response matrix, we found that not all of the measurements are needed to completely define a structure's dynamic characteristics. This property greatly simplifies the data acquisition process. It was shown that the frequency and damping could be extracted from any measurement in which the mode shape values are nonzero.

Mode shapes were discussed both in terms of generating a display and using them for further data analysis. Two mode shape scaling techniques were described to show that mode shape scaling is arbitrary as long as the inertial and elastic properties are preserved.

REFERENCES

1. Brown, D.L.; Allemang R.A.; Zimmerman, R.; Mergeay, M.; "Parameter Estimation Techniques for Modal Analysis" SAE Paper Number 790221, SAE Transactions, Volume 88, 1979, pp. 909 914.

2. Formenti, D.L.; "Analytical and Experimental Modal Analysis", May 1977 Modal Analysis Course, University of Cincinnati.

3. Klosterman, A.; McClelland, W.A.; "Combining Experimental and Analytical Techniques for Dynamic System Analysis"; Structural Dynamics Research Corporation, 1973, 20 pp.

4. Richardson, M.H.; "Modal Analysis Using Digital Test Systems"; Proceedings of Seminar on Understanding Digital Control and Analysis In Vibration Test Systems, May 1975.